Übungen 14-26

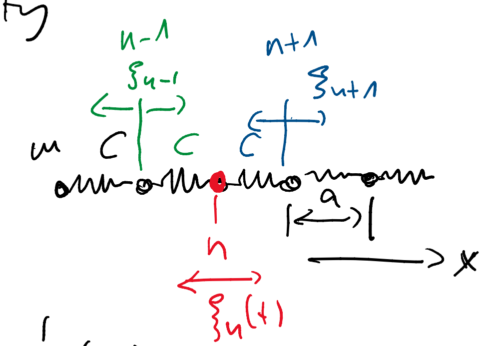
Mittwoch, 25. Oktober 2023 14:01

Problem 16 Elastic waves

$$(**) \quad \frac{\partial^2 \xi}{\partial t^2} = v^2 \frac{\partial^2 \xi}{\partial^2 x^2} \qquad v = \text{velocity of wave } \xi(x,t)$$

$$v = \sqrt{\frac{E}{\rho}} \qquad E = \text{Young modulus}, \quad \rho = \text{mass density}$$

Phonon



$$m\ddot{\xi}_n = C(\xi_{n-1} - \xi_n) + C(\xi_{n+1} - \xi_n)$$

$$\Rightarrow \quad = C(\xi_{n+1} + \xi_{n-1} - 2\xi_n) \qquad (*) \stackrel{!}{=} (**)$$

Taylor expansion of ξ around ξ_n:

$$\xi_{n+1} = \xi(x_{n+1}) = \xi(x_n) + \frac{\partial \xi}{\partial x}\Big|_{x=x_n} \cdot \underbrace{\overbrace{(x_{n+1} - x_n)}} + \frac{1}{2}\frac{\partial^2 \xi}{\partial x^2}\Big|_{x_n}(x_{n+1} - x_n)^2$$

$$= \xi(x_n) + \frac{\partial \xi}{\partial x}\Big|_{x_n} a + \frac{1}{2}\frac{\partial^2 \xi}{\partial x^2}\Big|_{x_n} a^2$$

$$\xi_{n-1} = \xi(x_{n-1}) = \xi(x_n) + \frac{\partial \xi}{\partial x}\Big|_{x_n} \underbrace{(x_{n-1} - x_n)} + \frac{1}{2}\frac{\partial^2 \xi}{\partial x^2}\Big|_{x_n}(x_{n-1} - x_n)^2$$

$$= \xi(x_n) + \frac{\partial \xi}{\partial x}\Big|_{x_n}(-a) + \frac{1}{2}\frac{\partial^2 \xi}{\partial x^2}\Big|_{x_n}(-a)^2$$

$$m\ddot{\xi}_n = C[\xi_{n+1} + \xi_{n-1} - 2\xi_n] \qquad \Delta\xi \sim \mathcal{O}\left(\xi \frac{a}{\lambda}\right)$$

$$m\ddot{\xi}_n = C\frac{\partial^2 \xi}{\partial x^2} a^2 \quad \Leftrightarrow \quad \ddot{\xi} = v^2 \frac{\partial^2 \xi}{\partial x^2}$$

$$\Rightarrow \quad v^2 = \frac{C}{m} a^2 \quad \Rightarrow \quad v = \sqrt{\frac{C}{m}}\, a$$

"classical" $v = \sqrt{\frac{E}{\rho}} \Leftarrow$

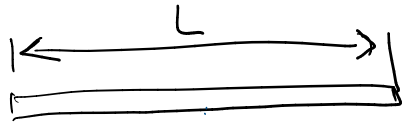
"Classical" $v = \sqrt{\frac{E}{\rho}} \leftarrow$

$$E = \rho v^2 = \frac{\cancel{m}}{a^3} \cdot \frac{C}{\cancel{m}} a^2 = \frac{C}{a} = E$$

$$E = \frac{\cancel{m}}{A \cdot \cancel{a}} \cdot \frac{C}{\cancel{m}} a^2 = \frac{Ca}{A}$$

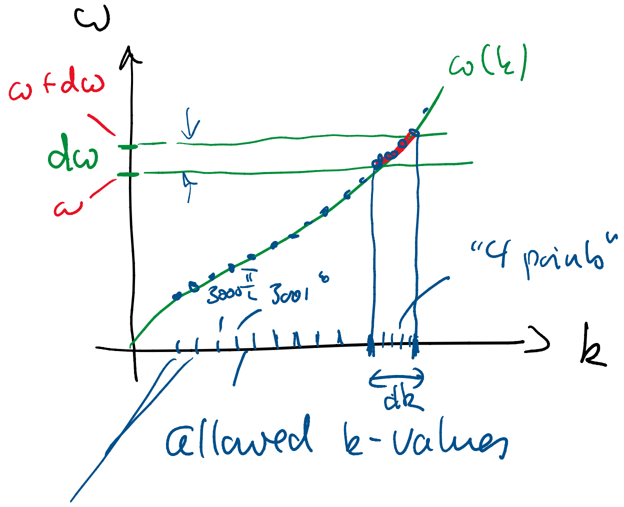
$\Rightarrow$ valid for $\lambda \gg a$ ($\rightarrow$ Debye-model!)

Problem 20: Density of states continuous medium



$\lambda \gg a$

dispersion relation $\omega(k)$



DOS $S(\omega)\,d\omega$ = # states in $[\omega, \omega + d\omega]$

$S(\omega) = 4$

$\Delta k =$

$k = \frac{2\pi}{\lambda} =$

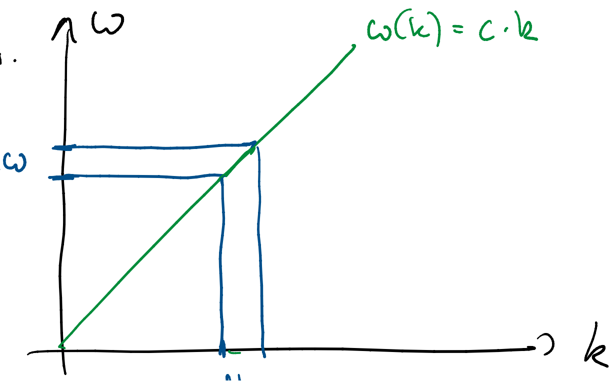
Standing wave $L = \frac{\lambda}{2}$

$$\Delta k = \frac{2\pi}{\lambda} = \frac{2\pi}{2L} = \frac{\pi}{L} = \frac{\pi}{N \cdot a} \qquad k_n = n \cdot \frac{\pi}{L}$$

discrete unit cell a $\Rightarrow$ $L = N \cdot a$

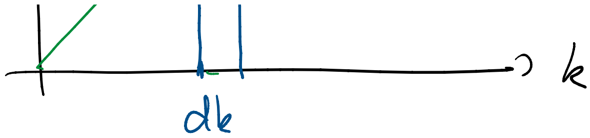
cont. medium

c = const.



$\omega = c \cdot k$

$$dk = \frac{dk}{d\omega} \cdot d\omega = \frac{1}{d\omega/dk}\, d\omega = \frac{d\omega}{c}$$



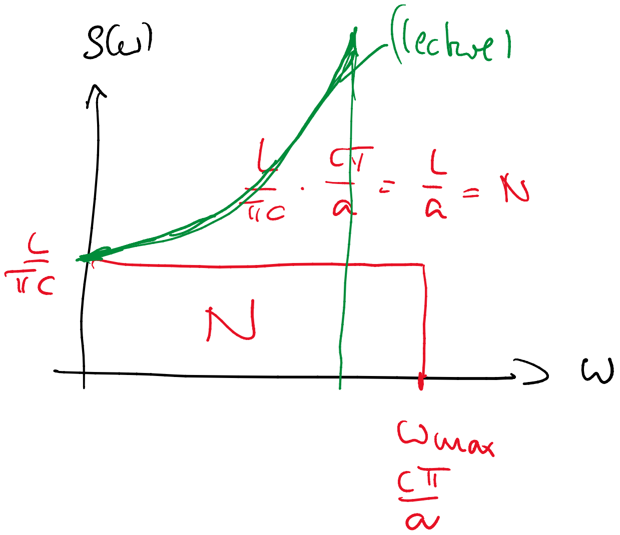
$= \frac{d\omega}{c}$

DOS $\quad \rho(\omega)\, d\omega = n_k \cdot dk = n_k \cdot \frac{d\omega}{c}$

density of k-values

$\Delta k = \frac{\pi}{L} \Rightarrow n_k = \frac{1}{\Delta k} = \frac{L}{\pi}$

$\Rightarrow \rho(\omega) = n_k \frac{1}{c} = \frac{L}{\pi c}$

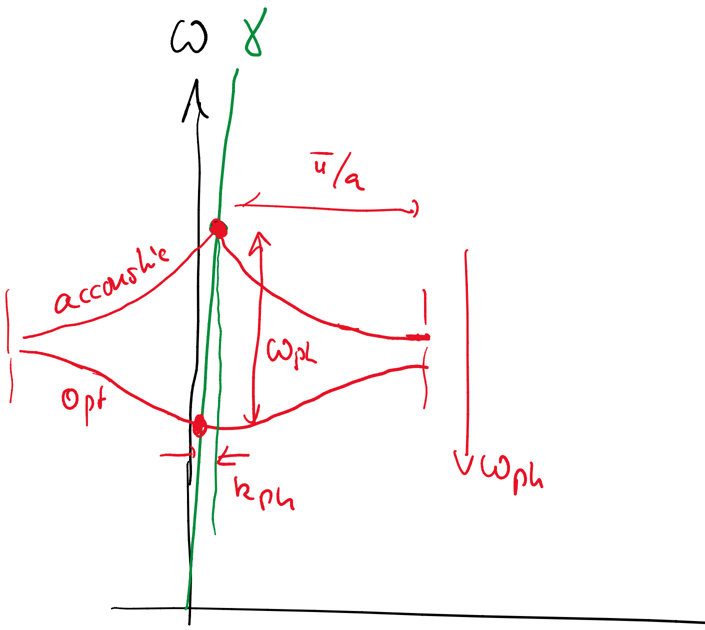


$k_{max} = \frac{\pi}{a} = \frac{N\pi}{L}$

$\omega_{max} = k_{max} \cdot c = \frac{c\pi}{a}$

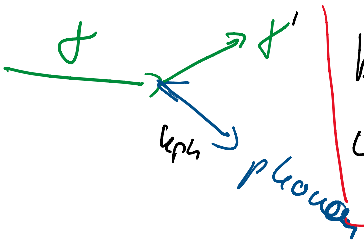
$\tilde{\rho}(\omega) = \frac{2N}{\pi (\tilde{\omega}_{max}^2 - \omega^2)^{1/2}}$

Problem 22: Inelastic scattering by light



γ: $\omega = c k$, $c = 3 \times 10^8\, m/s$

phonon: $c_s \sim 4000\, m/s$



$k_\gamma' = k_\gamma + k_{ph}$

$\omega_\gamma' = \omega_\gamma - \omega_{ph}$

optical phonons (dipole moment!)

$\hbar\omega_\gamma \approx 100\,meV \Rightarrow$ infrared

$\hbar\omega_{ph} \sim 10..\ 100\,meV$ $\quad$ ($k_B T \sim 300K$ $25\,meV$)

Afg. 19: Phononen in 2D NaCl

Mittwoch, 25. Oktober 2023 14:02