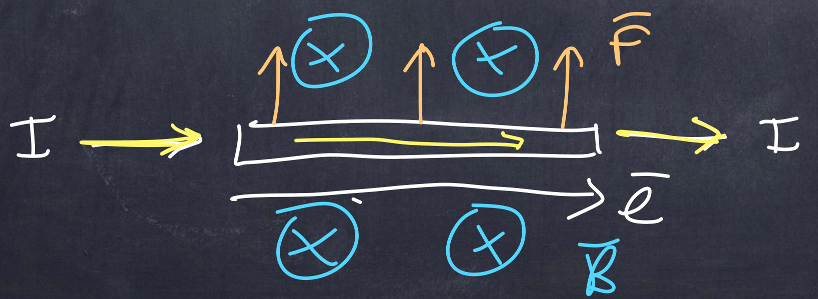
PHY117 HS2023

Week 10, Lecture 2
Nov. 22nd, 2023
Prof. Ben Kilminster

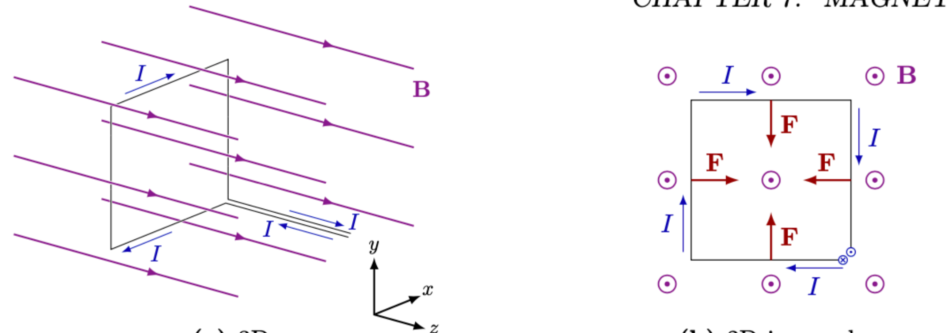
yesterday:



$F = BI\ell$

length of wire

what about a loop of current?



(a) 3D. (b) 2D in xy plane.

Figure 7.9: Rectangular current loop in an external, uniform magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$.

No net force, no net torque

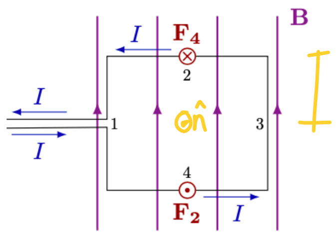


Figure 7.10: Rectangular current loop in an external magnetic field **B**.

Here, there is torque $\bar{\tau} = \bar{r} \times \bar{F}$

Segments 1+3 are parallel to $\bar{B}$, so no force, no torque.

Segment 2: $F_2 = BI\ell_2$, $\tau_2 = \frac{\ell_1}{2} BI\ell_2 \hat{x}$

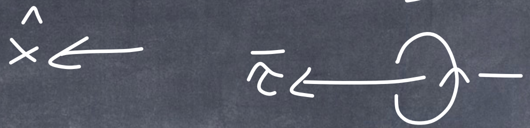
segment 4: $F_4 = BI\ell_2$, $\tau_4 = \frac{\ell_1}{2} BI\ell_2 \hat{x}$

The loop will twist from the torque

(Notice $\hat{n}$ of loop $\perp \bar{B}$)

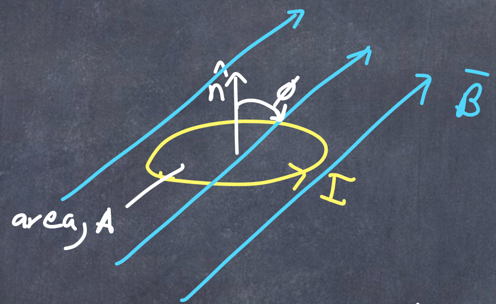
Total torque = $\tau_2 + \tau_4 = \frac{l_1}{2} B I l_2 + \frac{l_1}{2} B I l_2 = I(l_1 l_2) B = IAB\hat{x}$

area



If loop $\hat{n}$ is at an angle with respect to $\bar{B}$, then in general

$$\boxed{\bar{\tau} = IA\hat{n} \times \bar{B} = IAB\sin\phi}$$



ϕ : is the angle from $\hat{n}$ to $\bar{B}$

$\hat{n}$: normal direction $\perp$ to the plane of the loop.

Torque is always in the direction that aligns $\hat{n}$ & $\bar{B}$

we can define the magnetic moment of the loop as $\mu = IA$

and the $\bar{\mu}$ vector

$$\boxed{\bar{\mu} = (IA)\,\hat{n}}$$

then

$$\boxed{\bar{\tau} = \bar{\mu} \times \bar{B}}$$

(a) Magnetic moment of a current loop in a uniform magnetic field.

(b) Right-hand rule for the magnetic moment of a current loop.

Figure 7.11: Magnetic moment.

N loops

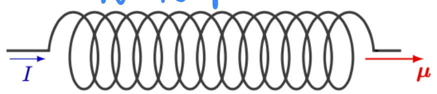


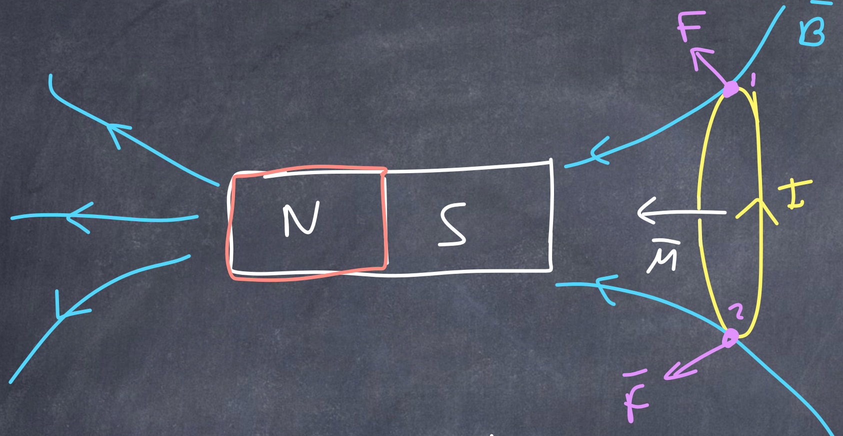
Figure 7.12: Magnetic moment of a solenoid with N windings.

$$\bar{\mu} = N(IA)\,\hat{n}$$

The potential energy of a current loop in $\bar{B}$-field is $\boxed{U = -\bar{\mu}\cdot\bar{B}}$ + constant

we set the constant so that when $\bar{\mu}$ is $\parallel$ to $\bar{B}$, then $U = 0$.

What if the magnetic field is non-uniform?

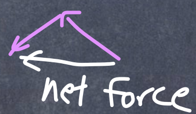


Consider 2 opposite points

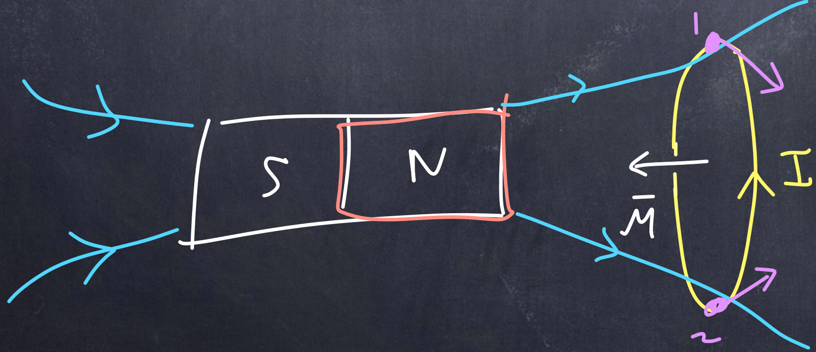
1: $\bar{F}_1 = q\bar{v}\times\bar{B}$ $\quad \left.\begin{array}{l}\bar{v} = \otimes \\ \bar{B} = \swarrow\end{array}\right\} \bar{F}: \nwarrow$

2: $\bar{F}_2$: $\left.\begin{array}{l}\bar{v} = \odot \\ \bar{B} = \nwarrow\end{array}\right\} \bar{F}: \swarrow$

The net force is towards the magnet.



(vertical components cancel out)

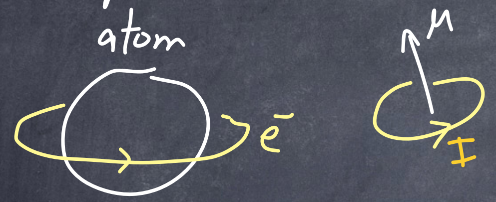


1: $\bar{F}$: $\left.\begin{array}{l}\bar{v} = \otimes \\ B = \nearrow\end{array}\right\} \bar{F} = \searrow$

2: $\bar{F}$: $\left.\begin{array}{l}\bar{v} = \odot \\ B = \searrow\end{array}\right\} \bar{F} = \nearrow$

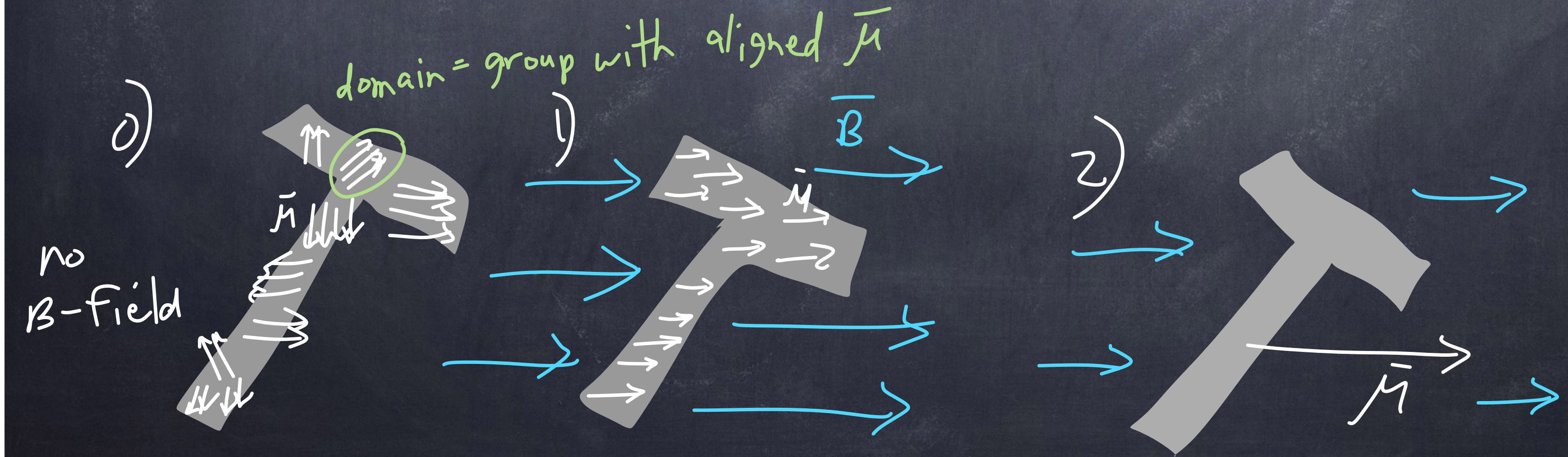
The net force is away from the magnet

Electrons and atoms can be thought of as spinning electric charges.



Each atom has a magnetic moment, $\bar{\mu}$.

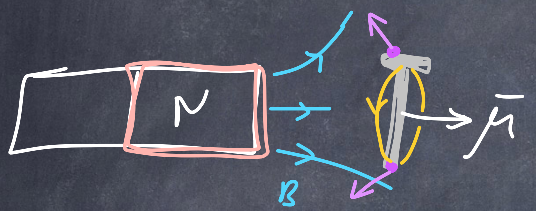
This helps us understand why an unmagnetized nail is attracted to a magnet, both the N + S side. This happens in a few steps:



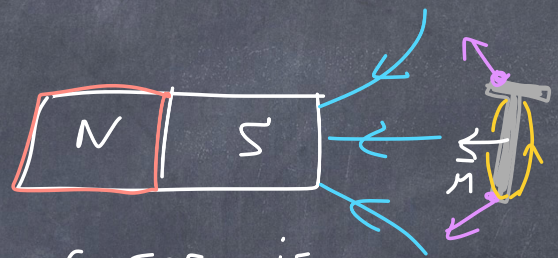
Why is nail attracted to N + S poles of magnet?

First, nail is magnetized in direction of field.

Second, divergent field causes a force of attraction



$\left.\begin{array}{l}\bar{V}=0\\ B=\nearrow\end{array}\right\}\ \bar{F}=\nwarrow$

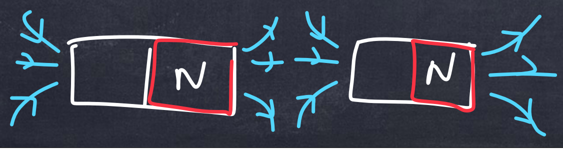


$\left.\begin{array}{l}\bar{V}=\otimes\\ B=\swarrow\end{array}\right\}\ F=\nwarrow$

The net force in both cases is toward the magnet.

No force [In a constant magnetic field

This is also why two magnets attract

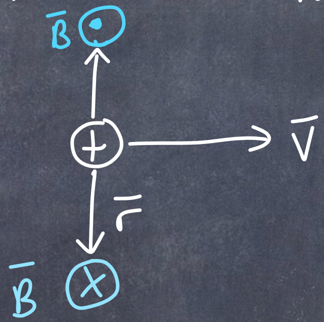


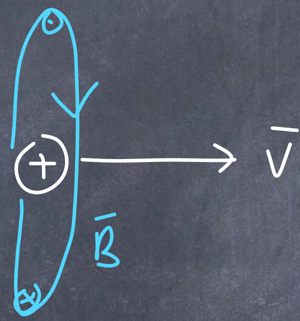
So far, we know: $\bar{F}_B = q\bar{v} \times \bar{B}$ $\qquad \bar{F} \perp \bar{v}, \bar{B}$

$\bar{F}_B = I\bar{\ell} \times \bar{B}$ $\qquad \bar{F} \perp \bar{\ell}, \bar{B}$

Now: A moving charge $\oplus \longrightarrow \bar{v}$ generates its own magnetic field.

The direction of $\bar{B}$ is $\bar{v} \times \bar{r}$





The magnetic field loops around the direction of motion.

The magnitude of B decreases like $\frac{1}{r^2}$

$$\boxed{\bar{B} = \frac{\mu_0}{4\pi}\,\frac{q\bar{v}\times\hat{r}}{r^2}}$$

$$[T] \qquad \left[\frac{T\cdot m}{A}\right] \qquad \frac{[C]\left[\frac{m}{s}\right]}{[m^2]} \qquad 1$$

$\bar{B}$ caused by a moving charge.

μ_0: permeability of free space

vacuum

$$\mu_0 = 4\pi\,E{-7}\ \frac{T\cdot m}{A}$$

$$[A] = \left[\frac{C}{s}\right]$$

For a current, the form is

$$\boxed{d\bar{B} = \frac{\mu_0}{4\pi}\,\frac{I\,d\bar{l}\times\hat{r}}{r^2}}$$

Biot-Savart law:
Integrate to solve for any shaped wire in a B-field.

we won't do any exercises with Biot-Savart law.

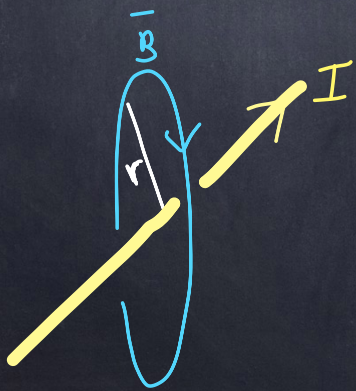
$$\bar{B} = \int_l \frac{\mu_0}{4\pi}\,\frac{I\,d\bar{l}\times\hat{r}}{r^2}$$

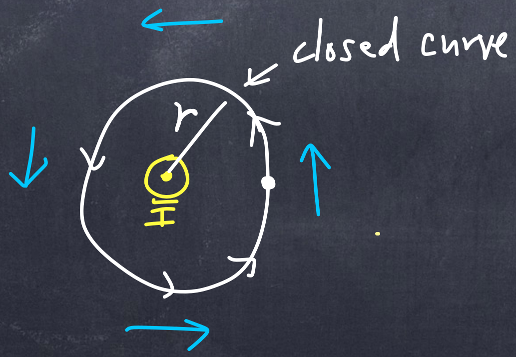
However, for simple configurations of current, there is an easier way. (like Gauss' Law)

Ampere's Law

$$\boxed{\oint_{\substack{\text{closed}\\ \text{curve}}} \bar{B}\cdot d\bar{l} = \mu_0 I_c}$$

I_c : current passing through the closed curve.





we pick a curve where $\bar{B} \parallel \bar{l}$

$$\oint_c \bar{B}\cdot d\bar{l} = \mu_0 I_c$$

$$\downarrow$$

$$B\oint_c dl = \mu_0 I$$

$$\oint_c dl = 2\pi r$$, the circumference of a circle

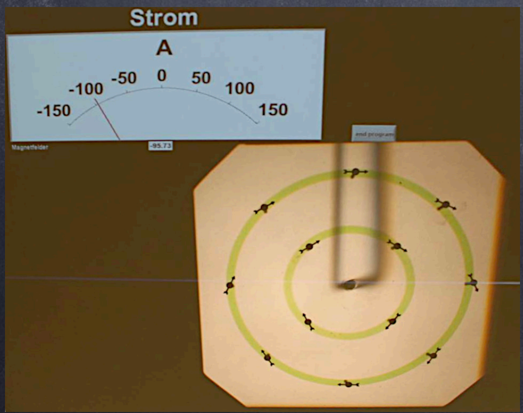
$$\downarrow$$

$$B(2\pi r) = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

we see here:

$$B \propto \frac{1}{r} \qquad B \propto I$$



Using Ampere's law on a solenoid:

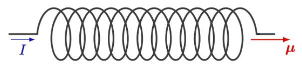
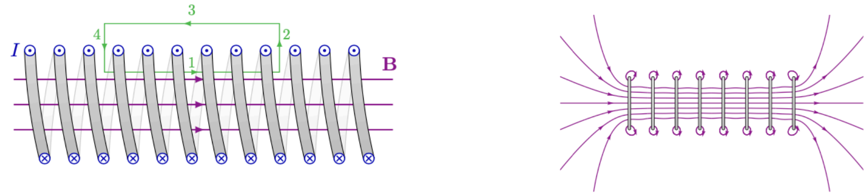


Figure 7.12: Magnetic moment of a solenoid with N windings.

8.2. AMPÈRE'S LAW 89



(a) Using Ampère's law on a rectangular loop. **(b)** Realistic field of a solenoid.

Figure 8.6: Magnetic field due to a solenoid.

sides 1 & 3 have length, x

$$n = \frac{N \text{ loops}}{\text{length}}$$

$$I_c = (nx) I$$

Then $\oint_c \bar{B} \cdot d\bar{l} = \mu_0 I_c$

$$\mu_0 n x I$$

$$\underbrace{\quad}_{\substack{Bx \\ \bar{B} \parallel d\bar{l}}}^{1} + \underbrace{\quad}_{\substack{0 \\ \bar{B} \perp d\bar{l}}}^{2} + \underbrace{\quad}_{\substack{0 \\ \bar{B} \cong 0}}^{3} + \underbrace{\quad}_{\substack{0 \\ \bar{B} \perp d\bar{l}}}^{4}$$

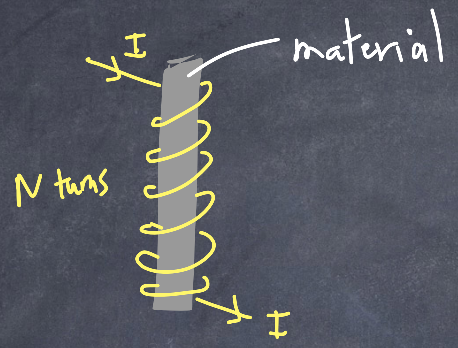
$$B\cancel{x} = \mu_0 n \cancel{x} I$$

$$\boxed{B = \mu_0 n I = \mu_0 \frac{N}{l} I}$$

magnetic field in a hollow Solenoid.

If there is a material inside,

$$B = \mu n I \quad \text{where } \mu = \mu_0 \kappa$$

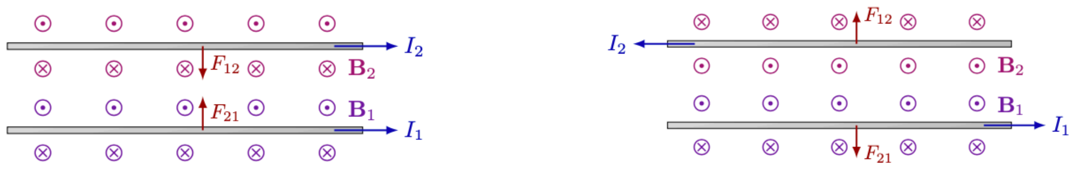


Here, κ is the relative permeability

material	κ $\left(\frac{\mu}{\mu_0}\right)$
air	1.000 000 37
water	0.99999 2
copper	0.999 994
pure iron (99.95%)	200 000
iron 99.8%	5000

A current I_1 produce a magnetic field $B_1 = \dfrac{\mu_0 I_1}{2\pi r}$

Another current I_2 will feel a force from B_1: $F = B_1 I_2 \ell$

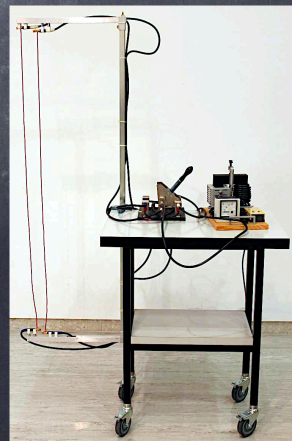


(a) Parallel current. **(b)** Anti-parallel current.

Figure 8.7: Magnetic force between current-carrying wires.

$$F_{12} = \frac{\mu_0 I_1 I_2 \ell}{2\pi r}$$

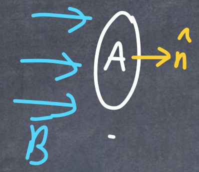
also $\bar{F}_{12} = -\bar{F}_{21}$



attractive or repulsive!

Magnetic flux :

For a loop $\perp$ to $\bar{B}$-field



we can quantify the $\bar{B}$-field by

$$\boxed{\phi_m = BA}$$

A : area

where ϕ_m is known as the magnetic flux

If $\hat{n}$ is not $\parallel$ to $\bar{B}$, then

$$\boxed{\phi_m = \bar{B}\cdot\hat{n}\,A = BA\cos\theta}$$

units are weber :

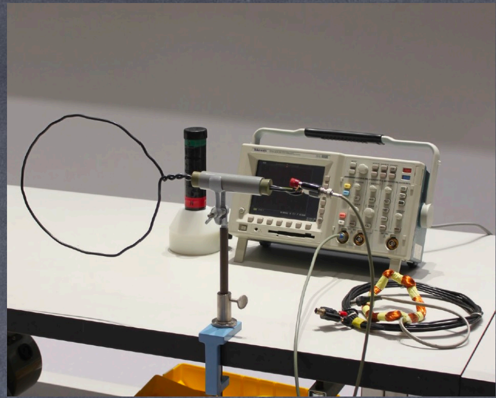
$$1\,[Wb] = 1\,[T\cdot m^2]$$

If magnetic flux changes, an electric field will be produced. The electric field produces an Emf

$$\boxed{\underset{\substack{\uparrow \\ \text{Voltage} \\ \text{supplied}}}{\mathcal{E}} = \oint_C \bar{E}\cdot d\bar{l} = -\frac{d\phi_m}{dt}}$$

Known as Faraday's Law.

Notice $[V] = \left[\frac{Wb}{s}\right]$



$\phi_m = \bar{B} \cdot \hat{n} A$

can be changed by

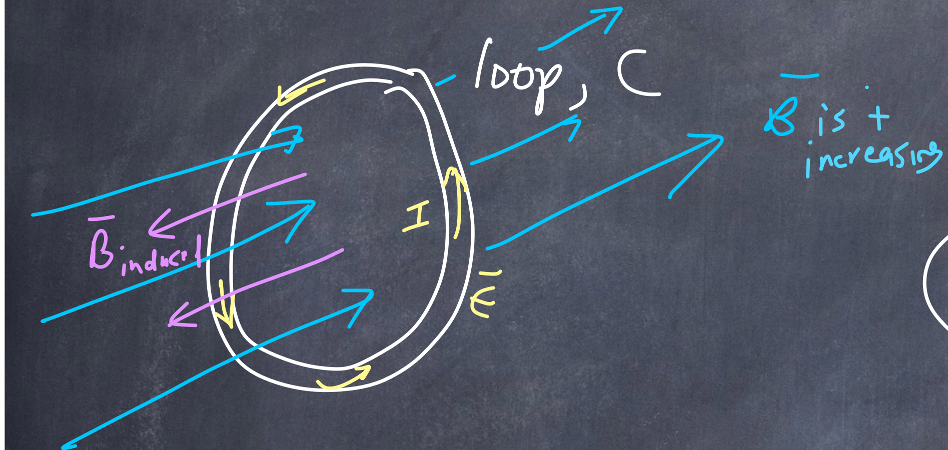
changing B or A or $\hat{n}$!

If magnetic flux changes, an electric field will be produced. The electric field produces an Emf. This electric field means that a current is produced. But a current produces a magnetic field! What?

$$\boxed{\text{Lenz's Law: "The induced Emf and induced current are in such a direction so as to oppose the change that produces them."}}$$

This means:

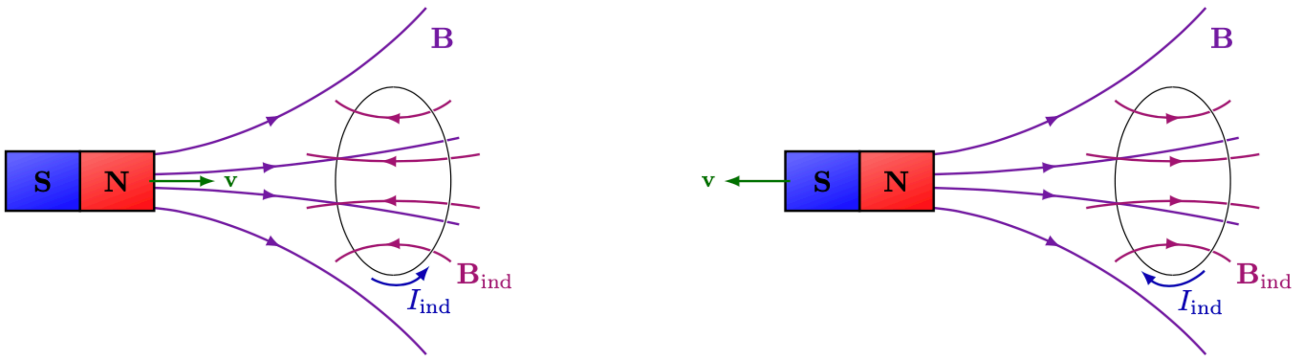
1) A moving magnet induces magnets in the opposite direction.



If we increase $\bar{B}$, I is produced.

(But in opposite direction)

$\bar{B}_{induced}$ opposes changes in $\bar{B}$



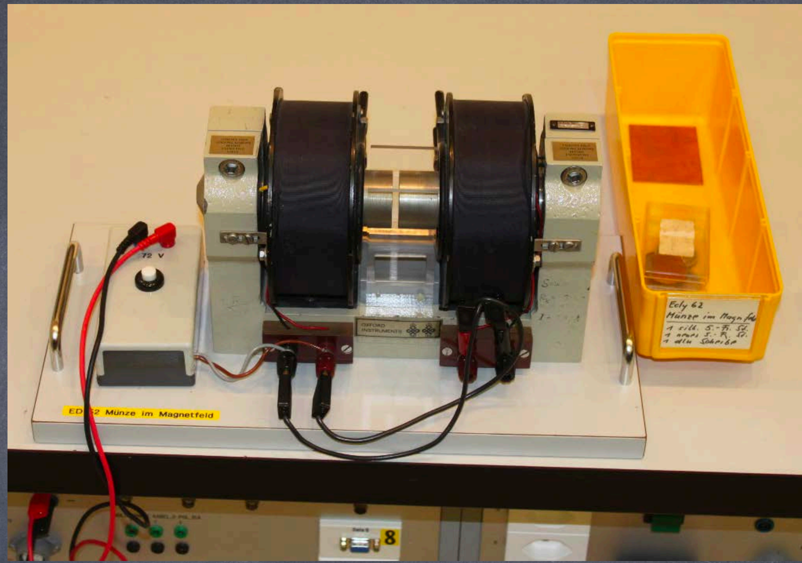
(a) Field moving toward the loop.

(b) Field moving away from the loop.

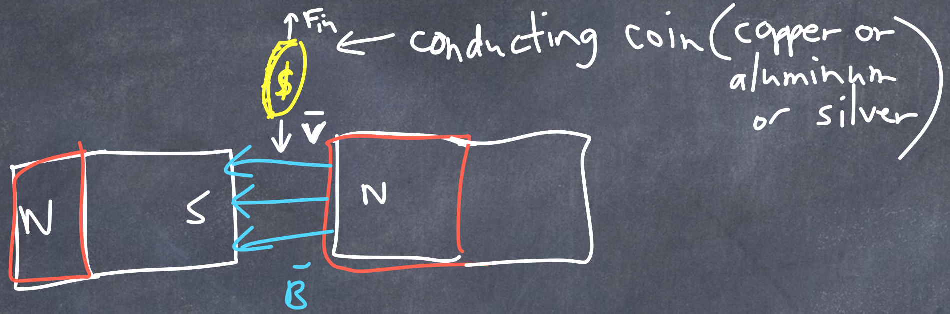
Figure 8.8: The magnetic field $\mathbf{B}$ of a moving bar magnet will induce a current I_{ind} in a conducting loop and therefore a magnetic field $\mathbf{B}_{ind}$.

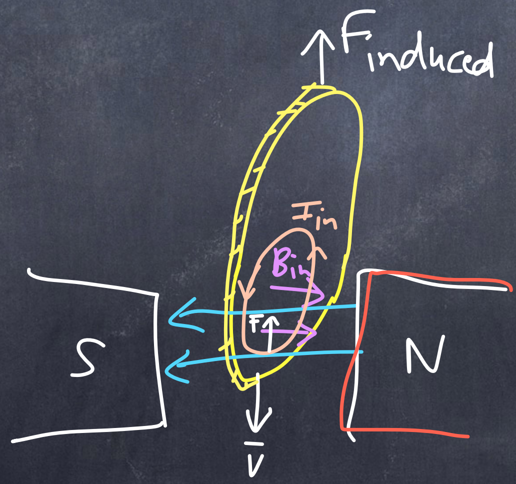
If $\bar{B}$ increases, $\bar{B}_{ind}$ is opposite $\bar{B}$

If B decreases, $\bar{B}_{ind}$ is in the same direction as $\bar{B}$



Dropping a conductor through a magnet.





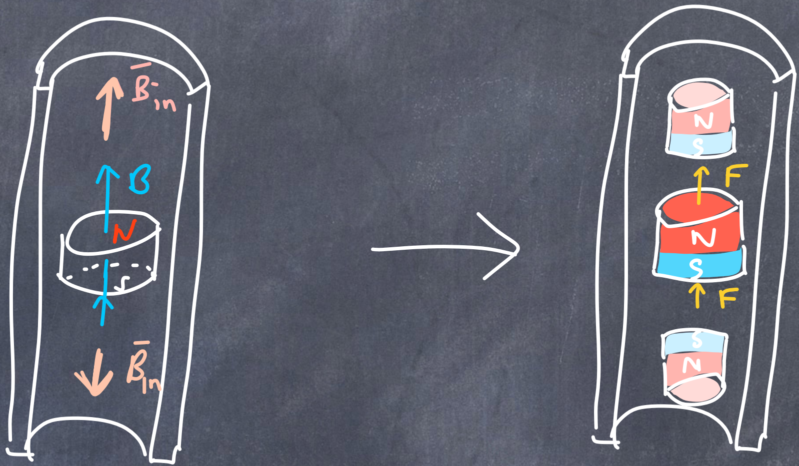
we call induced currents "Eddy currents"

cutout prevents Eddy currents

As magnet falls:

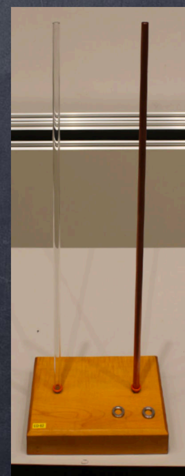
$\bar{B}$ decreases above magnet

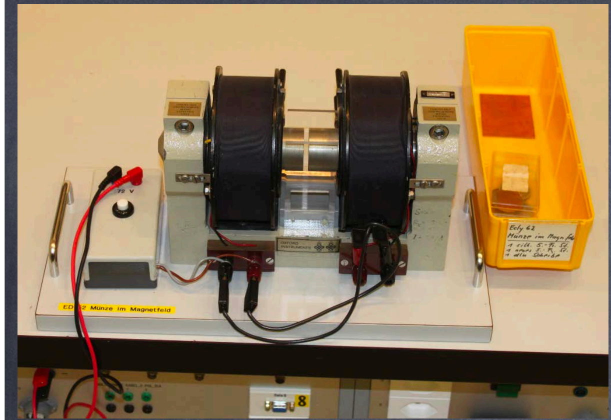
$\bar{B}_{in}$ is same direction as $\bar{B}$



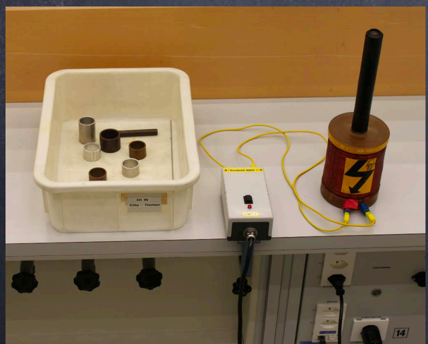
$\bar{B}$ increases below the magnet

$\bar{B}_{induced}$ opposite of $\bar{B}$





Dropping a conductor in a magnet



turning a conductor into an opposing magnet

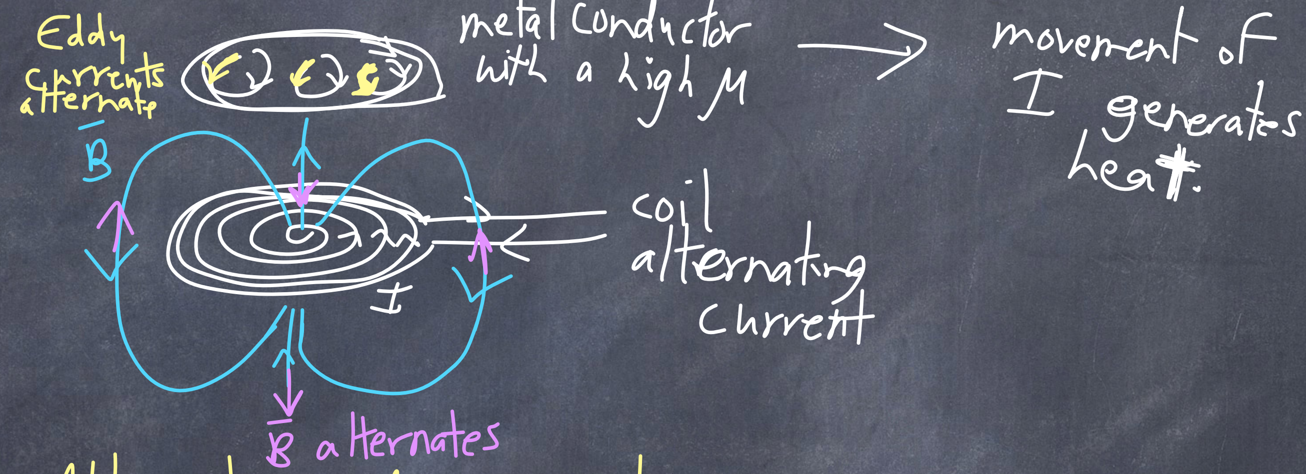
cutout prevents Eddy currents

dropping magnet in a conductor

Summary of magnetic field concepts:

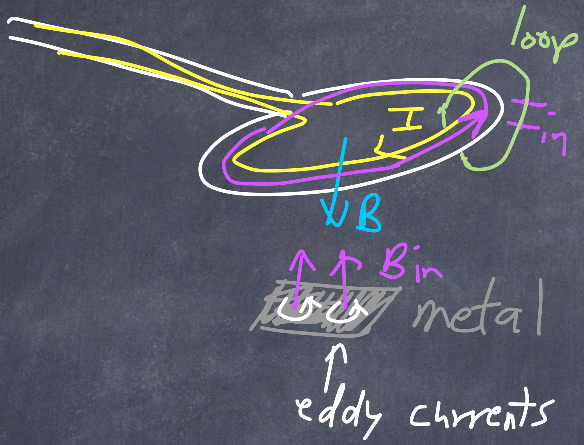
1) A moving electric charge may feel a force from a magnetic field.
2) A moving electric charge generates its own magnetic field. (A changing electric field produces a magnetic field.)
3) A changing magnetic field generates electric currents that produce an opposing magnetic field.

Induction stove uses Eddy currents:



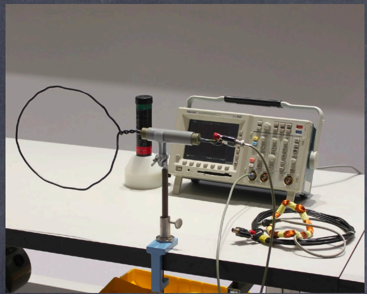
Alternating eddy currents generate heat in a conductor (Joule heating)

Metal detector uses Eddy currents



I_{in} is generated in opposite direction, tends to decrease current in metal detector.

Metal detector searches for currents in opposite directions.



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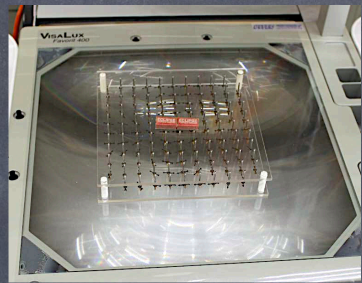

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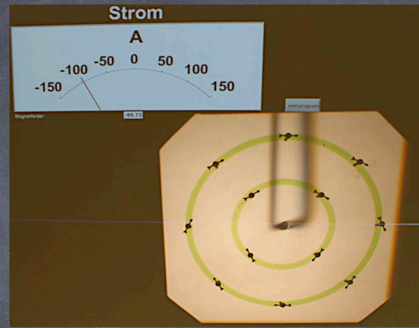

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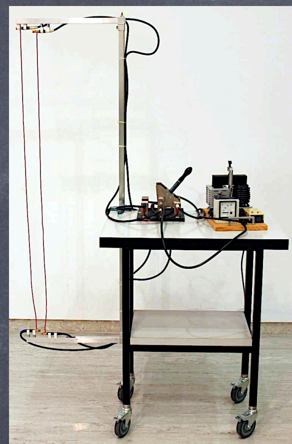

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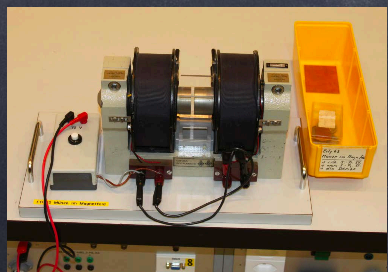

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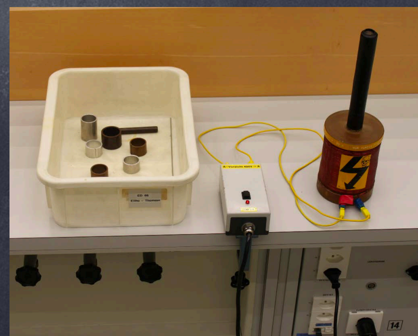

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